On the Maximum Performance in Opportunistic Routing

Llorenç Cerdà-Alabern and Amir Darehshoorzadeh
Univ. Politècnica de Catalunya
Computer Architecture Dep.
Barcelona, Spain
Email: {llorenc,amir}@ac.upc.edu

Vicent Pla
Univ. Politècnica de Valencia
Communications Dep.
València, Spain
Email: vpla@dcom.upv.es

***Abstract*—In recent years there has been a growing interest in Opportunistic Routing as a way to increase the capacity of wireless networks by exploiting its broadcast nature. By contrast to traditional uni-path routing, in opportunistic routing the nodes overhearing neighbor's transmissions can become candidates to forward the packets towards the destination.**

In this paper we address the question: What is the maximum performance that can be obtained using opportunistic routing? To answer this question we use an analytical model that allows to compute the optimal position of the nodes, such that the progress towards the destination is maximized. We use this model to compute bounds to the minimum expected number of transmissions that can be achieved in a network using opportunistic routing.

***Index Terms*—wireless networks; opportunistic routing; maximum performance; analytical model.**

I. Introduction

In this paper we study the maximum gain that can be obtained using Opportunistic Routing (OR). Previous works have studied OR selecting the OR candidates on a given network topology, and comparing the efficiency with the traditional uni-path routing. The efficiency is measured in terms of the expected number of transmissions from the source to the destination. Therefore, we shall refer to gain as the relative difference of the expected number of transmissions required with OR with respect to the traditional uni-path routing.

We address the question: What is the maximum gain that can be obtained using OR? More specifically, we are interested to answer this question when the maximum number of candidates per node is limited. To answer this question we need to choose a network where the nodes are optimally located so that at each transmission the progress towards the destination is maximized. To do so, we shall assume that we have a formula for the delivery probability between the nodes at a distance d, $p(d)$. More specifically, we shall consider that no collisions occur (there is only one node transmitting at a time), and thus, that $p(d)$ is given by the radio propagation model of the network.

An expression to compute the expected number of transmissions in OR (that, like in [1], we refer to as EAX) has been obtained by several authors (see e.g. [1], [2]). That expression is recursive and has a non linear dependence on the delivery probability between the nodes. Therefore, even if $p(d)$ is known, EAX may not give a feasible way to derive the optimal position, and thus, the maximum OR gain. We solve this problem by computing the optimal position of the nodes maximizing the progress towards the destination when OR is used. We show that this approach allows to derive a set of equations that can be solved numerically in order to compute the optimal positions of the candidates. Additionally, maximizing the progress towards the destination permits to establish a lower bound on the expected number of transmissions in OR, and thus, a maximum bound on the OR gain.

The remainder of this paper is organized as follows. In section II we study the positions of the candidates that yield the maximum progress per transmission, and the results are used in Section III to derive bounds for the expected number of transmission necessary to reach the destination. Section IV introduces the propagation model that we have used in the numerical experiments presented in Section V. In Section VI we describe a procedure to construct a network using the maximum progress distances. The expected number of transmissions in this network yields a tight upper bound of the optimal value. Section VII compares the maximum progress distances with the optimal distances. In Section VIII a practical method is proposed to achieve a performance close the optimum one with a much lower number of nodes. Finally, Section IX surveys the related work and concluding remarks are given in Section X.

II. Optimal Positions of Candidates

We study the position of the candidates in order to maximize the progress towards the destination. The ingredients of our model are: The maximum number of candidates per node n, and the formula for the delivery probability at a distance d, $p(d)$, which we suppose to be the same for all the nodes. Assume that the destination is far from a generic test node for which we are looking the candidates. Clearly, the optimum candidates will be located over the segment between the test node and the destination (see figure 1).

Let $\{c_1, c_2, \cdots c_n\}$ be the ordered set of candidates of the generic test node (c_n the highest priority, and c_1 the least one), and d_i the distance from the test node to the candidate c_i (see figure 1). We assume that a coordination protocol exist

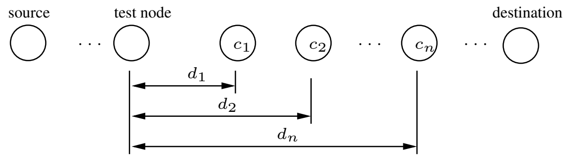

Fig. 1. Test node and its candidates.

among the candidates, such that the highest priority candidate receiving the packet will forward the packet (if it is not the destination), while the other nodes will simply discard it. Assume that $p(d_i)$ is the delivery probability from the test node to the candidate c_i, and let Δ_n be the random variable equal to the distance reached after one transmission shot. Clearly,

$$\begin{aligned} \mathrm{E}[\Delta_n] = &\ d_n\, p(d_n) + \\ &\ d_{n-1}\, p(d_{n-1})\, (1 - p(d_n)) + \\ &\ \cdots + d_1\, p(d_1) \prod_{i=2}^{n} (1 - p(d_i)) \\ = &\ d_n\, p(d_n) + (1 - p(d_n))\, \mathrm{E}[\Delta_{n-1}]. \end{aligned} \tag{1}$$

We are interested in looking for the value $d_n \in (d_{n-1}, \infty)$ that maximizes equation (1). Note that this value maximizes also the function

$$f(x) = (x - a)\, p(x) \tag{2}$$

where $a = \mathrm{E}[\Delta_{n-1}]$. Notice that $f(a) = 0$ and $f(x)$ is increasing in the neighborhood of a. We shall assume that the delivery probability $p(x)$ is differentiable and $\lim_{x\to\infty} x\, p(x) = 0$, which make plausible to further assume that the function $f(x)$ is quasi-concave in $x \in (a, \infty)$, having a unique critical point equal to its global maximum in this interval. This condition holds e.g. for the shadowing model we use to assess $p(d)$ (see section IV). Additionally, since $\mathrm{E}[\Delta_{n-1}] < d_{n-1} < d_n$, we can reduce the optimization domain to $d_n \in (d_{n-1}, \infty)$. Under these conditions we can compute the distances d_i, $i = 1, \cdots, n$ that maximize (1) by solving:

$$\frac{\partial \mathrm{E}[\Delta_i]}{\partial d_i} = 0, \quad d_i \in (d_{i-1}, \infty),\ i = 1, \cdots, n$$

which gives the set of equations:

$$\begin{aligned} p(d_i) + (d_i - \mathrm{E}[\Delta_{i-1}])\, p'(d_i) = 0, \\ d_i \in (d_{i-1}, \infty),\ i = 1, \cdots, n \end{aligned} \tag{3}$$

where $\mathrm{E}[\Delta_0] = 0$ and $d_0 = 0$. Note that using equation (3) we can compute d_1 by solving $p(d_1) + p'(d_1)\, d_1 = 0$, after which we can compute d_2 and so on until d_n. We shall refer to these distances as the *maximum progress distances*. In the sequel we shall refer to them as $d_1, \cdots, d_n$, and denote the expected number of transmissions given by equation (1) using these distances as $\mathrm{E}[\Delta_n^*]$. Note also that a consequence of equation (3) is that the maximum progress distances for the already existing candidates do not change if we decide to add a new candidate to the candidate set.

III. Maximum Performance of OR

In this section we investigate the performance of OR in terms of the expected number of transmissions to send a packet from the source to the destination. To do so, we define τ_n to be the random variable equal to the number of transmissions required to send a packet from the source to the destination using n candidates per node. We are thus interested in obtaining bounds to $\mathrm{E}[\tau_n]$.

A. A Lower Bound with Infinite Candidates

We first derive a result that will be useful in the bounds derived afterwards. Assume an infinitely dense network where the nodes can choose an infinite number of candidates. Assume further that there is not limitation on the minimum delivery probability that live links can have. Let τ_∞ be the random variable equal to the number of transmissions required to send a packet from the source to the destination in such network. With these assumptions, some node as close to the destination as we want can receive the packet with probability 1 (we can choose a region arbitrarily close to the destination that contains an infinite number of candidates). Therefore, if the destination does not receive the packet after it is firstly transmitted by the source, some candidate arbitrarily close to it will receive it an relay it to the destination with just one more transmission, and thus, $\tau_\infty = 2$. Let D be the distance between the source and the destination. From the previous discussion we conclude that:

$$\mathrm{E}[\tau_\infty] = p(D) + 2(1 - p(D)) = 2 - p(D). \tag{4}$$

B. A Lower Bound for the Expected Number of Transmissions

Assume a network with n candidates per node. Since $\mathrm{E}[\Delta_n^*]$ computed in section II using the maximum progress distances given by equations (3) is the maximum progress towards the destination after every transmission shot, we have that the expected number of transmissions to send a packet from the source to the destination ($\mathrm{E}[\tau_n]$) is lower bounded as follows

$$\mathrm{E}[\tau_n] \geq \frac{D}{\mathrm{E}[\Delta_n^*]} \tag{5}$$

where D is the distance between the source and the destination.

The bound given by equation (5) will be tight as long as the distance D is large compared with d_n, and the nodes are located at the maximum progress distances. Clearly, when the nodes become closer than d_n to the destination, the optimal positions cannot be given by the maximum progress distances. In this case the highest priority candidate will be the destination. Thus, the distance of the most priority candidate will be the distance to the destination, and the optimal position of the other candidates should be computed taking the distances that minimize the EAX formula. In fact, this "boundary effect" will propagate to the position of the other nodes between the source and the destination, and their optimal positions may be slightly different than those obtained using the maximum progress distances (we shall investigate this in section VII). Nevertheless, the expected distance progress after each transmission could not be as high as the one obtained using the maximum progress distances, which guarantees that (5) is a lower bound.

We can use the result obtained for an infinite number of candidates to improve the bound given by (5). First, the expected number of transmissions cannot be less that the value given by equation (4). Therefore, we have that:

$$\mathrm{E}[\tau_n] \geq \max\left(2 - p(D),\ \frac{D}{\mathrm{E}[\Delta_n^*]}\right). \qquad (6)$$

The bound given by (6) can still be improved when $n > 1$ as we explain next. As we said before, when the nodes are closer than d_n to the destination, the position of the nodes cannot be the maximum progress distances. Therefore, using $\mathrm{E}[\Delta_n^*]$ as the progress in this region may be a coarse approximation. To estimate the progress in this region we note that before the packet reaches the destination, at least one node in the interval $[D - d_n, D)$ will receive it, because the furthest candidate of any node is at a distance d_n. We shall refer to the first node in this interval that receives the packet as $v(x)$, where x is the distance from this node to the destination (we assume that the source is located at 0, and the destination at D). Now, the number of transmissions from the source to $v(x)$ can be lower bounded by $(D-x)/\mathrm{E}[\Delta_n^*]$ (i.e. assuming the maximum progress), and the number of transmissions from $v(x)$ to the destination can be lower bounded assuming an infinite number of candidates between $v(x)$ and the destination (equation (4)). Adding both terms we have $\mathrm{E}[\tau_n|v(x)] \geq (D-x)/\mathrm{E}[\Delta_n^*] + 2 - p(x) = D/\mathrm{E}[\Delta_n^*] + 2 - p(x) - x/\mathrm{E}[\Delta_n^*]$. Thus, if we want a lower bound we must take x that minimizes $\mathrm{E}[\tau_n|v(x)]$ in the interval $x \in (0, d_n]$.

Summing up, we have that:

$$\mathrm{E}[\tau_n] \geq \begin{cases} \max\left(2 - p(D),\ \dfrac{D}{\mathrm{E}[\Delta_1^*]}\right),\ n = 1 \\ \max\left(2 - p(D),\ \dfrac{D}{\mathrm{E}[\Delta_n^*]} + \right. \\ \left. \inf\limits_{x \in (0,\, d_n]} \left\{2 - p(x) - \dfrac{x}{\mathrm{E}[\Delta_n^*]}\right\}\right),\ n > 1 \end{cases} \qquad (7)$$

C. An Upper Bound for the Gain

Let us denote by τ_n the number of transmissions when the candidates are optimally placed. In order to measure the improvement that can be reached using OR we define the *gain* (G_n) as the relative difference of the expected number of transmissions required with the OR with n candidates ($\mathrm{E}[\tau_n]$), with respect to the uni-path routing case. Note that OR with only 1 candidate per node is equivalent to uni-path routing. Therefore, we shall refer to the expected number of transmissions with uni-path routing as $\mathrm{E}[\tau_1]$, and thus:

$$G_n = \frac{\mathrm{E}[\tau_1] - \mathrm{E}[\tau_n]}{\mathrm{E}[\tau_1]} = 1 - \frac{\mathrm{E}[\tau_n]}{\mathrm{E}[\tau_1]}. \qquad (8)$$

Using the same intuition as in (5) we can write

$$\frac{D}{\mathrm{E}[\Delta_1^*]} \leq \mathrm{E}[\tau_1] \leq \frac{\lceil D/d_1 \rceil d_1}{\mathrm{E}[\Delta_1^*]} \qquad (9)$$

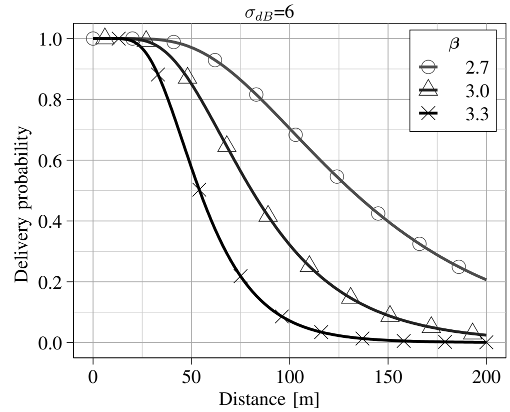

Fig. 2. Delivery probability versus distance for $\sigma_{dB} = 6$ dBs.

and using the lower bound for $\mathrm{E}[\tau_n]$ and the upper bound for $\mathrm{E}[\tau_1]$ it follows from (8) that

$$G_n \leq 1 - \frac{D/d_1}{\lceil D/d_1 \rceil} \frac{\mathrm{E}[\Delta_1^*]}{\mathrm{E}[\Delta_n^*]}. \qquad (10)$$

IV. Propagation Model

In order to assess the delivery probabilities we will assume that the channel impairments are characterized by a shadowing propagation model: The power received at a distance d ($P_r(d)$), in terms of the transmitted power (P_t) is given by:

$$P_r(d)|_{dB} = 10\ \log_{10}\left(\frac{P_t\, G_t\, G_r\, \lambda^2}{L\,(4\,\pi)^2\, d^\beta}\right) + X_{dB}. \qquad (11)$$

Where G_t and G_r are the transmission and reception antenna gains respectively, L is a system loss, λ is the signal wavelength (c/f, with $c = 3 \times 10^8$ m/s), β is a path loss exponent and X_{dB} is a Gaussian random variable with zero mean and standard deviation σ_{dB}.

Packets are correctly delivered if the received power is greater than or equal to `RXThresh`. Note that we shall not consider collisions in our model. Thus, the delivery probability at a distance d ($p(d)$) is given by:

$$\begin{aligned} p(d) &= Prob(P_r(d)|_{dB} \geq 10\ \log_{10}(\texttt{RXThresh})) = \\ &Q\left(\frac{1}{\sigma_{dB}} 10\ \log_{10}\left(\frac{\texttt{RXThresh}\ L\,(4\,\pi)^2\, d^\beta}{P_t\, G_t\, G_r\, \lambda^2}\right)\right) \end{aligned} \qquad (12)$$

where $Q(z) = \frac{1}{\sqrt{2\,\pi}} \int_z^\infty \mathrm{e}^{-y^2/2} dy$.

In our numerical experiments we have set the model parameters to the default values used by the network simulator (ns-2) [3], given in table I. Figure 2 depicts the delivery probability at a varying distance, for three values of the path loss exponent

TABLE I
Default ns values for the shadowing propagation model.

Parameter	Value
P_t	0.28183815 Watt
`RXThresh`	3.652×10^{-10} Watt
G_t, G_r, L	1
f	914 MHz

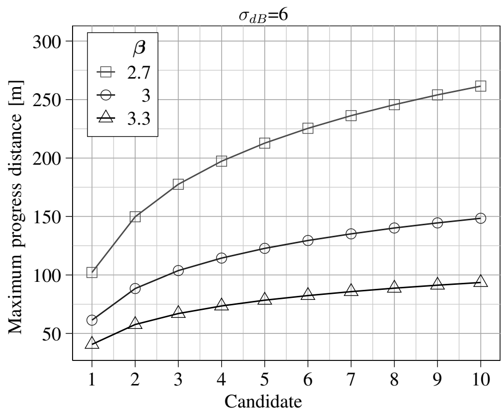

Fig. 3. Maximum progress distances for the candidates.

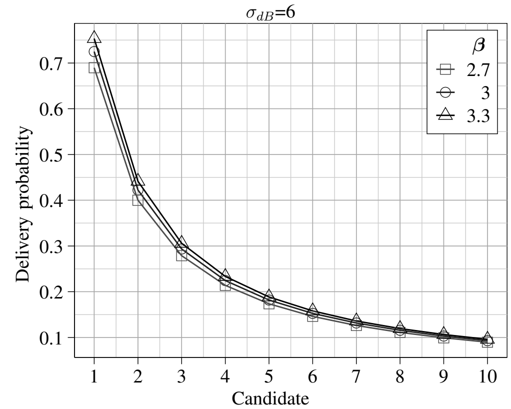

Fig. 4. Delivery probability to each candidate located at the maximum progress distances.

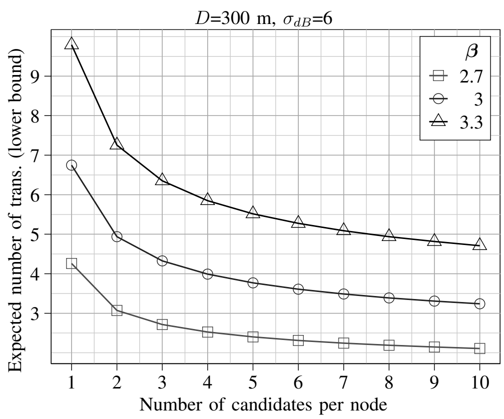

Fig. 5. Expected number of transmissions (lower bound).

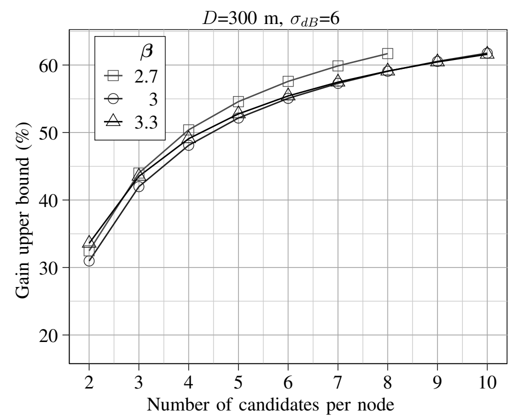

Fig. 6. Gain (upper bound).

(β) and a standard deviation $\sigma_{dB} = 6$ dBs. We shall use these values in the numerical results presented in later sections.

V. Numerical Results

We now give some numerical examples of the formulas derived in the previous sections. We shall assume that the delivery probability $p(d)$ is given by equation (12). Substituting $p(d)$ in the equations (3) and solving them numerically we obtain the maximum progress distances for the candidates shown in figure 3. There are three curves, that correspond to three values of the loss exponent of the propagation model: $\beta = 2.7$, $\beta = 3$ and $\beta = 3.3$. Note that the larger is β, the lower is the transmission range of the nodes, and thus, the shorter are the distance of the candidates.

Figure 4 shows the delivery probabilities obtained for the corresponding points shown in figure 3. It is interesting that the probabilities are very similar for all values of β. This fact could be use as a rule of thumb in the selection of candidates.

Finally, figure 5 depicts the lower bound of the expected number of transmissions (equation (7)) for a distance $D = 300$ m between the source and the destination, and figure 6 shows the corresponding upper bound to the gain (equation (10)). As we shall see in section VI, the lower bounds given by equation (7) are very tight. Consequently, the gains that can be obtained using OR are close to the upper bounds depicted in figure 6. These figures show that the highest gain increase occurs when we move from 1 to 2 candidates (approximately 30% of gain). After which the gain increases approximately up to 60% with 10 candidates. However, implementing an OR protocol with a high number of candidates is difficult, and possibly will introduce large signaling overhead and duplicated transmissions that would prevent to reach such large gains. This motivates that selecting a maximum number of candidates per node equal to 2 or maybe 3 is possibly a sensible choice.

VI. Quasi Optimal OR Network

In this section we compute an upper bound for the expected number of transmissions by computing EAX in a network where the candidates are positioned using the maximum progress distances computed as in section II. Note that not all the candidates can be located using these distances, since for some nodes the distance to the destination can be shorter than the distance to the candidate. For these nodes we will use the destination and its closest neighbors located between the node and the destination as candidates. Since these candidates, at least, are not located at the optimum positions, the expected number of transmissions computed for such network will be an upper bound to the minimum expected number of transmissions that can be achieved using OR. We shall refer

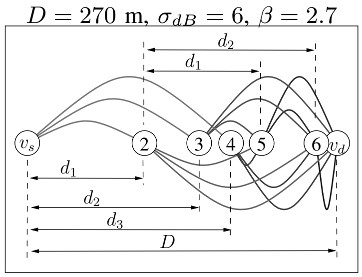

Fig. 7. Quasi Optimal OR network with a maximum of 3 candidates per node.

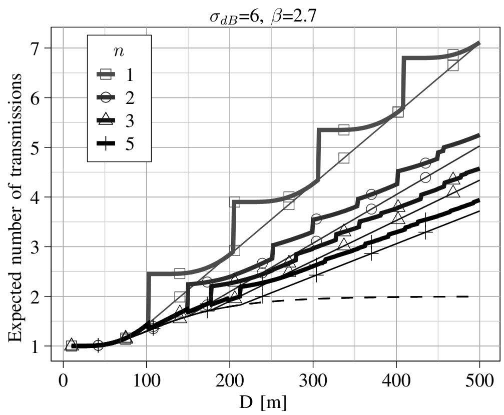

Fig. 8. Lower and upper bounds (thin and thick lines respectively) of the minimum expected number of transmissions achievable with OR for $n = 1$, 2, 3 and 5 maximum number of candidates. The dashed line corresponds to infinite number of candidates.

to such network as *Quasi Optimal OR (QOO) network*.

Figure 7 depicts an example of a network with 3 candidates per node build using these rules. The source is v_s and the destination is v_d. Nodes 2, 3 and 4 are located at the maximum progress distances from v_s: d_1, d_2 and d_3 respectively. Nodes 5 and 6 are located at the maximum progress distances from node 2: d_1 and d_2 respectively. Since v_d is closer from node 2 than d_3, v_d is taken as the third candidate of node 2. Since node 6 is at a distance d_1 from node 3, and v_d is closer from this node than d_2 and d_3, the candidates of node 3 are nodes 5, 6 and v_d. Likewise it is done for the other nodes.

Figure 8 shows the expected number of transmissions varying the distance D between the source and the destination for a QOO network build as explained before. The curves shown in the figure have been obtained using a maximum number of candidates per node equal to 1, 2, 3 and 5 (cfr. the numbers in the legend). Figure 9 shows the number of nodes that resulted in the QOO networks used to obtain the corresponding values of figure 8. In figure 8 we have also added the lower bounds of equation (7) (thin lines), and the lower bound for an infinite number of candidates given by equation (4) (dashed line).

The delivery probability of the links ($p(d)$) has been obtained using the shadowing model (equation (12)) with a path loss exponent $\beta = 2.7$. The expected number of transmissions has been obtained using the Markov chain that we have proposed in [4]. These values could have been obtained also using the recursive formula of the expected number of transmissions

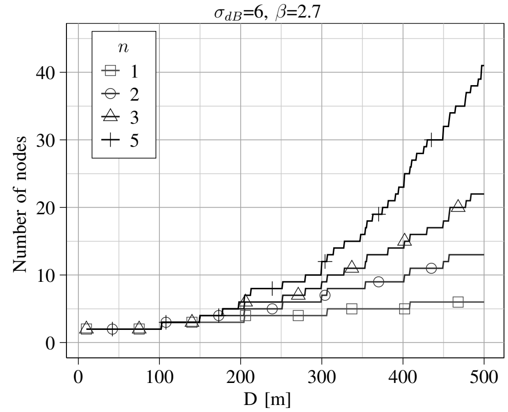

Fig. 9. Number of nodes of the quasi optimal OR networks used in figure 8

that has been proposed by several authors (see e.g. [1], [2]). However, we have noticed that solving the Markov chain was faster than using the recursive formula.

Figure 8 confirms that the lower bounds of the expected number of transmissions obtained with equation (7) are very tight, since they are very close to the upper bound obtained with the QOO network. Furthermore, this result seems to indicate that the maximum progress distances are very close to the optimum distances. We shall investigate this in the next section. Note that the discontinuities of the upper bound occur at the distances where a new node is added to the QOO network. E.g. in the scenario with 1 candidate, which occurs when the distance between the source and the destination (D) is a multiple of d_1.

VII. Validation

In the previous sections the *maximum progress distances* have been obtained and used to derive bounds, which are rather accurate approximations as well, of the performance of OR measured by the mean number of transmission required to reach the destination. For a network of finite length ($D < \infty$), the optimal distances of the candidates —in the sense of minimizing the mean number of transmissions required to reach the destination— are more complex to obtain and in general may not coincide with the *maximum progress distances*. In this section, we use a numerical approximation to estimate the optimal distances in a finite length network with the aim of empirically confirming some of the intuitions that have been applied previously, and provide a further insight into the optimal distances problem.

Let $V_n(x)$ be the minimum mean number of transmissions required to reach the destination that is at distance x from the source node, when a maximum of n candidates per node is used. We can write

$$V_n(x) = \min_{x_1 < \cdots < x_n} \left\{ \frac{1}{1 - \prod_{i=1}^n q(x_i)} \Big(1 + p(x_n) V_n(x - x_n) + \cdots + \prod_{i=2}^{n} q(x_i) p(x_1) V_n(x - x_1) \Big) \right\} \quad (13)$$

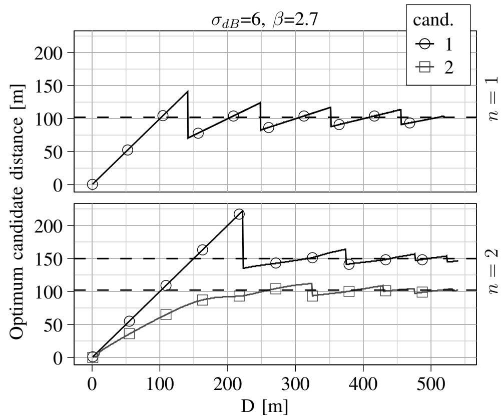

Fig. 10. Optimum distances of the candidates (d_i^*) with a maximum of $n = 1$ and $n = 2$ candidates per node (top and bottom respectively). The dashed lines are the maximum progress distances (d_i).

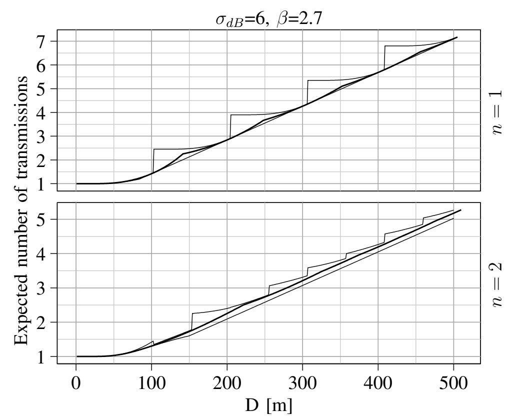

Fig. 11. Expected number of transmissions obtained with the optimum distances of figure 10, and its upper and lower bounds.

where $q(d) = 1 - p(d)$ and $V_n(0) = 0$. If the number of nodes between the source and the destination is less than n, then the destination and the intermediate nodes are taken as candidates. We shall refer as d_i^* to the optimal distances x_i that minimize equation (13).

We have solved the optimization problem of (13) in an approximate fashion by considering a discrete network (a finite number of nodes are evenly distributed between source and destination) and then performing an exhaustive optimum search. The network density, i.e., the number of nodes, have been increased until the minimum obtained does not vary significantly. Obviously, the exhaustive search becomes unfeasible as the maximum number of candidates, n, or the network size, D, grow. For this reason, we have limited this method to a maximum number of candidates equal to $n = 1$ and $n = 2$. Nevertheless, as we will see in the following, these two scenarios are enough for validation purposes.

Figure 10 compares the optimal distances (d_i^*) and the maximum progress distances (d_i) as functions of D. The optimum mean number of transmission obtained for the optimal distances are shown in Fig. 11 along with their corresponding lower and upper bounds.

We observe that the optimal distances converge to the maximum progress distances ($d_1 \approx 102\ m$ and $d_2 \approx 150\ m$) when D grows. It is also observed that while the maximum progress distances of the first candidate (d_1) are the same for different values of the maximum number of candidates, the optimum distances d_1^* are different for $n = 1$ and for $n = 2$, although they converge to the same value (that of the maximum progress distance, d_1, as we said before).

Notice that, as expected, when $n = 1$ and D is a multiple of d_1, the optimal distance equals the maximum progress distance ($d_1^* = d_1$) and the lower bound of $V_1(D)$ turns out to yield an exact value. Also, as it has been predicted, the lower bound for $V_1(D)$ is tighter than that for $V_2(D)$. On the other hand, in both cases ($n = 1, 2$) when D grows the shape of $V_n(D)$ tends to be a straight line whose slope is matched by that of the lower bound, i.e., by $1/\mathrm{E}[\Delta_n^*]$. Moreover, the shape of

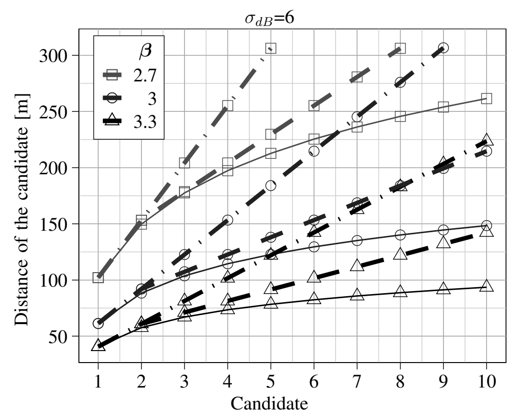

Fig. 12. Maximum progress distances of the candidates (thin lines) and its $d_1/4$ (dashed line) and $d_1/2$ (dot-dashed line) approximations.

$V_2(D)$ gets smooth more rapidly than $V_1(D)$ does. A similar observation can be made about the rate of convergence of the optimal distances to the maximum progress distances.

VIII. Sensitivity to Node Positions

The maximum progress distances computed in section II can be of practical interest in the design of a static network using OR. E.g the back-haul of a mesh network, or the position of the nodes in a sensor network. A first approach could be the Quasi Optimal OR Network described in the previous section. However, for such network the number of nodes increases nearly exponentially with the distance between the source and the destination, D, as shown in figure 9. In this section we look for positions of the nodes that, being close to their optimal values, allow reducing the number of nodes of the network.

Looking at the maximum progress distances obtained for different parameters of the propagation model (figure 3), we can observe that $d_2 \approx d_1 + d_1/2$ and $d_3 \approx d_1 + d_1/2 + d_1/4$. This suggest that a good compromise is positioning the nodes equally spaced at a distance $d_1/4$, choosing d_1 for the first candidate, $\hat{d}_2 = d_1 + d_1/2$ for the second, and $\hat{d}_i = d_1 + d_1/2 + (i-2) \times d_1/4$ for the candidates $i > 2$. Doing this way, a distance D would require a number of nodes $N \leq$

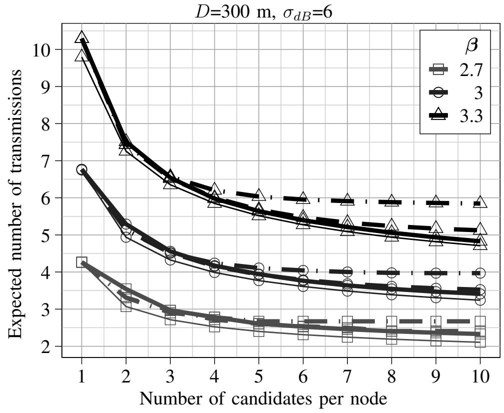

Fig. 13. Expected number of transmissions: (i) Lower bound (thin lines), (ii) using the QOO network of section VI (solid lines), (iii, iv) using, respectively, its $d_1/4$ (dashed line) and $d_1/2$ (dot-dashed line) approximations.

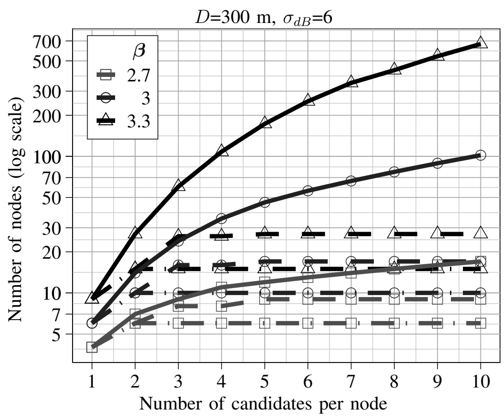

Fig. 14. Number of nodes of the networks used in figure 12 (i) using the QOO network of section VI (solid lines), (ii, iii) using its $d_1/4$ (dashed line) and $d_1/2$ (dot-dashed line) approximations.

$4 \cdot \lceil D/d_1 \rceil$. If only 2 candidates are going to be used, or if we wish to reduce further the number of nodes, a coarser approach would be positioning the nodes equally spaced at a distance $d_1/2$, choosing d_1 for the first candidate and $\hat{d}_i = d_1 + (i-1) \times d_1/2$ for the candidates $i > 2$. Doing this way, the required number of nodes would be $N \leq 2 \cdot \lceil D/d_1 \rceil$. We shall refer to these approximations as $d_1/4$ and $d_1/2$ respectively. Figure 12 shows the maximum progress distances computed as in section II and its $d_1/4$ and $d_1/2$ approximations.

Figures 13 and 14 show the sensitivity of the expected number of transmissions to the $d_1/4$ and $d_1/2$ approximations. As in section V, we have used a distance between the source and the destination $D = 300$ m, and three values of the loss exponent of the propagation model: $\beta = 2.7$, $\beta = 3$ and $\beta = 3.3$. For each value of β figure 13 shows four curves of the expected number of transmissions: (i) the lower bound computed as in section III (note that these curves are the same than those shown in figure 5); (ii) using the QOO network of section VI (solid lines); and (iii, iv) using its $d_1/4$ (dashed line) and $d_1/2$ (dot-dashed line) approximations. Figure 14 shows the number of nodes of the networks that where used to compute the expected number of transmissions for the corresponding cases (ii, iii, iv) of figure 13.

Figure 13 shows that the expected number of transmissions obtained for the QOO network is very close to the lower bound. Nevertheless, figure 14 shows that building the QOO network requires a high number of nodes. The maximum value is 665 nodes, obtained for $\beta = 3.3$ (where the nodes' coverage is the shortest) and 10 candidates per node. Figure 13 shows too that the expected number of transmissions obtained for the $d_1/4$ and $d_1/2$ approximations it is also very close to the lower bound. Only for $\beta = 3.3$ and more than 5 candidates per node the difference is noticeable. However, in figure 14 we can see that the number of nodes using the $d_1/4$ and $d_1/2$ approximations is enormously reduced (e.g. it is 27 and 15 nodes respectively for the $d_1/4$ and $d_1/2$ approximations in the same scenario for which 665 nodes are used with the QOO network).

We conclude that choosing the position of the 2 candidates closest to the sender near to their optimal positions, is the most critical in order to minimize the expected number of transmissions. Consequently, what we have called $d_1/2$ approximation may be a sensible rule of thumb in the design of the node positions in a static network using OR routing.

IX. Related Work

The majority of previous studies that evaluate the performance of opportunistic routing do not use analytical methods, instead they resort to simulations or empirical measurements [5], [6], [7], [8], [9], [10]. On the other hand, most of the works are devoted to the selection of the candidates, the way of acknowledging packet reception and how to prevent, or at least reduce, duplicate transmissions. Many of them prioritize the candidates according to route costs based on single-path metrics such as ETX [5]. In [1], [11] Zhong et al. proposed the *expected anypath transmission* (EAX) as a new metric for OR that generalizes the single-path metric ETX [12], and proposed a candidate selection and prioritization algorithm based on it. In [13], [14] an algebraic approach is applied to study the interaction of OR routing algorithms and routing metrics. The authors of [10] proposed an utility-based model for opportunistic routing and claimed that for the optimal solution it is necessary to search all loop free routes from source to destination. They proposed both optimal and heuristic solutions for selecting the candidates according to their utility function.

Baccelli et al. [15] aimed at quantifying and optimizing the potential performance gains of opportunistic routing strategies compared with classical routing schemes. Their analysis was under the assumptions of Aloha-based MAC layer. In [16] Shah et al. presented a framework to model OR for low loaded sensor networks. In [17], they also explored the performance of opportunistic routing for different node densities, channel quality and traffic rates, and compared it to geographic routing with simulation. They also identified optimal points for the duty cycle of nodes that minimize the power consumption.

In [18], the authors proposed an analytical approach for studying the potential gain of opportunistic routing in wireless networks. They provided bounds for the performance gain that can be achieved in opportunistic routing using shadowing and fading propagation models. However, in their study they assumed an unlimited number of candidates over a network topology with the nodes distributed over the plane according to a spatial Poisson distribution. In [19] they extend their work by using directional antennas and different radio propagation models and spatial node distributions.

X. Conclusions

In this paper we have derived the equations that yield the distances of the candidates in Opportunistic Routing (OR) such that the per transmission progress towards the destination is maximized. We have called them as the *maximum progress distances*. The only ingredient to obtain these distances is the law for the delivery probability between nodes as a function of distance. An important consequence of our derivation is that the the maximum progress distances for the already existing candidates do not change if we decide to add a new candidate to the candidate set.

Based on these maximum progress distances, we have proposed a lower bound to the expected number of transmissions needed to send a packet using OR. The lower bound has proven to be very tight.

By modeling the delivery probabilities with a shadowing propagation model we obtained numerical results showing that the expected number of transmissions can be reduced up to a 30% with only 2 candidates, whereas in order to reduce it another 30% the number of candidates has to be increased up to 10.

We have constructed a quasi optimum OR network locating the nodes and their candidates at the maximum progress distances whenever possible. Solving the expected number of transmissions in these networks we have confirmed that our lower bound is very tight. We have further validated these results by building a dense network and computing the optimal distances of the candidates by an exhaustive optimum search. We have seen that the optimal distances of the candidates converge rapidly to the maximum progress distances as the length of the network increases.

Finally, we have investigated the sensitivity to the position of the candidates. We have concluded that choosing the distance of the first two candidates near to their optimal positions, is the most critical in order to minimize the expected number of transmissions. Based on this result we have used the maximum progress distances to provide a rule of thumb for placing the nodes in a static network using OR. Compared to the optimal layout, this method will slightly increase the average number of transmissions while the total number of nodes required is reduced enormously. This can be of practical interest in the design of the back-haul of a mesh network, or in the positioning of the nodes in a sensor network.

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